***SOLUTION Section* 3.8 – Taylor and Maclaurin Series**

***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***







 

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by: 

***Solution***

























***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by : 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

















***Exercise***

Find out the third term of the Maclaurin series for the following function. 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find out the third term of the Maclaurin series for the following function. 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find out the third term of the Maclaurin series for the following function. 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find out the third term of the Maclaurin series for the following function. 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***

















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***

















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***

















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***















***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***











***Exercise***

Find the *n*th Maclaurin polynomial for the function 

***Solution***













***Exercise***

Find the Maclaurin series for: 

***Solution***

















***Exercise***

Find the Maclaurin series for: 

***Solution***













***Exercise***

Find the Maclaurin series for: 

***Solution***



















***Exercise***

Find the Maclaurin series for: 

***Solution***







***Exercise***

Find the Maclaurin series for: 

***Solution***







***Exercise***

Find the Maclaurin series for: 

***Solution***









***Exercise***

Find the Maclaurin series for: 

***Solution***







***Exercise***

Find the Maclaurin series for: 

***Solution***









***Exercise***

Find the Maclaurin series for: 

***Solution***









***Exercise***

Find the Maclaurin series for: 

***Solution***

 



***Exercise***

Find the Maclaurin series for: 

***Solution***





 



***Exercise***

Find the Maclaurin series for: 

***Solution***













***Exercise***

Find the Maclaurin series for: 

***Solution***

 







***Exercise***

Find the Maclaurin series for: 

***Solution***











***Exercise***

Find the Maclaurin series for: 

***Solution***









***Exercise***

Find the Maclaurin series for: 

***Solution***







***Exercise***

Finding Taylor and Maclaurin Series generated by : 

***Solution***















***Exercise***

Finding Taylor and Maclaurin Series generated by : 

***Solution***















***Exercise***

Finding Taylor and Maclaurin Series generated by :



***Solution***





















***Exercise***

Finding Taylor and Maclaurin Series generated by : 

***Solution***





















***Exercise***

Find the Taylor series of the functions, where is the series representation valid? 

***Solution***

Let 







***Exercise***

Find the Taylor series of the functions, where is the series representation valid? 

***Solution***

Let 









***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***

Let 











***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***







 



***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***









***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***











***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***









 

***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***









 



***Exercise***

Find the Taylor series of the functions, where is the series representation valid?



***Solution***

















***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***













***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***















***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***













***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***













***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***















***Exercise***

Find the *n*th Taylor polynomial centered at *c* for the function 

***Solution***











***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |



***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |



***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |



***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |



***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |





***Exercise***

Find the *n*th−order Taylor polynomial centered at *c* for the function 

***Solution***

 





 





 





 









***Exercise***

Find the sums of the series 

***Solution***





***Exercise***

Find the sums of the series 

***Solution***







***Exercise***

Find the sums of the series 

***Solution***





***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***

The Maclaurin series for  is 

By the *Root test*:





At ,

the series is  which diverges

At ,

the series is  which diverges

The *interval* of convergence is the real line 

***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***

The Maclaurin series for  is



By the *Root test*:





At ,

the series is  which diverges

At ,

the series is  which diverges absolutely (*harmonic*)

The *interval* of convergence is the real line 

***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***

The Maclaurin series for  is



By the *Root test*:





At ,

the series is  which diverges

At ,

the series is  which diverges absolutely (*harmonic*)

The *interval* of convergence is the real line 

***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***

The Maclaurin series for  is



By the *Root test*:





At ,

the series is  which diverges

At ,

the series is  which diverges absolutely (*harmonic*)

The interval of convergence is the real line 

***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***







Thus, the Maclaurin series for 





 





At ,

the series is  which diverges absolutely

At ,

the series is  which diverges

The *interval* of convergence is the real line 

***Exercise***

Use the geometric series , for , to determine the Maclaurin series and the interval of convergence for the following function



***Solution***













 







At ,

the series is  which converges

At ,

the series is  which diverges

The *interval* of convergence is the real line 

***Exercise***

The limit  that is the relative error in the approximation 

Approaches zero as *n* increases. That is *n*! grows at a rate comparable to . This result, known as Stirling’s Formula, is often very useful in applied mathemmatics and statistics. Prove it by carrying out the following steps.

1. Use the identity  and the increasing nature of ln to show that if ,



And hence that 

1. If , show that





1. Use the Maclaurin series for  to show that





and therefore that  is decreasing and  is increasing. Hence conclude that  exists, and that



***Solution***

1. 







1. If , then























1. 





 







 ***Geometric series*** 













These inequalities imply that  is decreasing and  is increasing.

Thus  is bounded below by  

So  exists.

Since  , we have



 exists.

***Exercise***

Suppose you want to approximate  to within  of the exact value.

1. Use a Taylor polynomial for  centered at 0.
2. Use a Taylor polynomial for  centered at 125.
3. Compare the two approaches. Are they equivalent?

***Solution***



1. 

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |













1. 

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |











1. Both the results from part (*a*) and (*b*) are the same since they are just shifting.

***Exercise***

Consider the function



1. Use the definition of the derivative to show that 
2. Assume the fact that  for *k* = 1, 2, 3, …. Write the Taylor series for  centered at 0.
3. Explain why the Taylor series for  does not converge to  for 

***Solution***

1. 









Let 







 ***√***

1. ***Given***: 

Since the Taylor series centered at 0 has only one term  and  (derivaties are equal to 0).

Therefore; the Taylor series is zero.

1. It does not converge to  because when , 

***Exercise***

Teams *A* and *B* go into sudden death overtime after playing to a tie. The teams alternate possession of the ball and the first team to score wins. Each team has a  chance of scoring when it has the ball, with Team *A* having the ball first.

1. The probability that Team *A* ultimately wins is . Evaluate this series.
2. The expected number of rounds (possessions by either team) required for the overtime to end is . Evaluate this series.

***Solution***

1. 

It is a *Geometric* series with , then







1. Using the series 





Let 





